

Accelerated Mathematics 3

This is a course in precalculus and statistics, designed to prepare students to take AB or BC Advanced Placement Calculus. It includes rational, circular trigonometric, and inverse trigonometric functions; basic trigonometric identities and the laws of sines and cosines; sequences and series; polar and parametric equations; vectors; the central limit theorem and confidence intervals. (*Prerequisite: Accelerated Mathematics 2 or Mathematics 3.*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

ALGEBRA

Students will explore characteristics of various functions, understand and use concepts of trigonometric functions, investigate and apply sequences and series, and use parametric and polar equations to represent functions and curves.

MA3A1. Students will explore rational functions.

- a. Investigate and explain characteristics of rational functions, including domain, range, zeros, points of discontinuity, intervals of increase and decrease, rates of change, local and absolute extrema, symmetry, asymptotes, and end behavior.
- b. Find inverses of rational functions, discussing domain and range, symmetry, and function composition.
- c. Solve rational equations and inequalities analytically, graphically, and by using appropriate technology.

MA3A2. Students will use the circle to define the trigonometric functions.

- a. Define and understand angles measured in degrees and radians, including but not limited to 0° , 30° , 45° , 60° , 90° , their multiples, and equivalences.
- b. Understand and apply the six trigonometric functions as functions of general angles in standard position.
- c. Find values of trigonometric functions using points on the terminal sides of angles in the standard position.
- d. Understand and apply the six trigonometric functions as functions of arc length on the unit circle.
- e. Find values of trigonometric functions using the unit circle.

MA3A3. Students will investigate and use the graphs of the six trigonometric functions.

- a. Understand and apply the six basic trigonometric functions as functions of real numbers.
- b. Determine the characteristics of the graphs of the six basic trigonometric functions.
- c. Graph transformations of trigonometric functions including changing period, amplitude, phase shift, and vertical shift.

- d. Apply graphs of trigonometric functions in realistic contexts involving periodic phenomena.

MA3A4. Students will investigate functions.

- Compare and contrast properties of functions within and across the following types: linear, quadratic, polynomial, power, rational, exponential, logarithmic, trigonometric, and piecewise.
- Investigate transformations of functions.
- Investigate characteristics of functions built through sum, difference, product, quotient, and composition.

MA3A5. Students will establish the identities below and use them to simplify trigonometric expressions and verify equivalence statements.

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

MA3A6. Students will solve trigonometric equations both graphically and algebraically.

- Solve trigonometric equations over a variety of domains, using technology as appropriate.
- Use the coordinates of a point on the terminal side of an angle to express x as $r \cos \theta$ and y as $r \sin \theta$.
- Apply the law of sines and the law of cosines.

MA3A7. Students will verify and apply $\frac{1}{2}ab \sin C$ to find the area of a triangle.

MA3A8. Students will investigate and use inverse sine, inverse cosine, and inverse tangent functions.

- Find values of the above functions using technology as appropriate.
- Determine characteristics of the above functions and their graphs.

MA3A9. Students will use sequences and series

- a. Use and find recursive and explicit formulae for the terms of sequences.
- b. Recognize and use simple arithmetic and geometric sequences.
- c. Investigate limits of sequences.
- d. Use mathematical induction to find and prove formulae for sums of finite series.
- e. Find and apply the sums of finite and, where appropriate, infinite arithmetic and geometric series.
- f. Use summation notation to explore series.
- g. Determine geometric series and their limits.

MA3A10. Students will understand and use vectors.

- a. Represent vectors algebraically and geometrically.
- b. Convert between vectors expressed using rectangular coordinates and vectors expressed using magnitude and direction.
- c. Add and subtract vectors and compute scalar multiples of vectors.
- d. Use vectors to solve realistic problems.

MA3A11. Students will use complex numbers in trigonometric form.

- a. Represent complex numbers in trigonometric form.
- b. Find products, quotients, powers, and roots of complex numbers in trigonometric form.

MA3A12. Students will explore parametric representations of plane curves.

- a. Convert between Cartesian and parametric form.
- b. Graph equations in parametric form showing direction and beginning and ending points where appropriate.

MA3A13. Students will explore polar equations.

- a. Express coordinates of points in rectangular and polar form.
- b. Graph and identify characteristics of simple polar equations including lines, circles, cardioids, limaçons, and roses.

DATA ANALYSIS AND PROBABILITY

Students will organize, represent, investigate, interpret, and make inferences from data, using the central limit theorem and the standard normal distribution. Students will apply the Central Limit Theorem to calculate confidence intervals for a population mean using data from large samples. Students will use sample data and confidence intervals to draw conclusions about populations.

MA3D1. Using simulation, students will develop the idea of the central limit theorem.

MA3D2. Using student-generated data from random samples of at least 30 members, students will determine the margin of error and confidence interval for a specified level of confidence.

MA3D3. Students will use confidence intervals and margins of error to make inferences from data about a population. Technology is used to evaluate confidence intervals, but students will be aware of the ideas involved.

Terms/Symbols: rational function, trigonometric function, period, amplitude, phase shift, cotangent, secant, cosecant, series, recursive formula, vector, parametric form, polar form, confidence interval, level of confidence, central limit theorem, margin of error, standard deviation, confidence interval, correlation

Process Standards

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MA3P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

MA3P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

MA3P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

MA3P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

- c. Recognize and apply mathematics in contexts outside of mathematics.

MA3P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

Reading Standard Comment

After the elementary years, students are seriously engaged in reading for learning. This process sweeps across all disciplinary domains, extending even to the area of personal learning. Students encounter a variety of informational as well as fictional texts, and they experience text in all genres and modes of discourse. In the study of various disciplines of learning (language arts, mathematics, science, social studies), students must learn through reading the communities of discourse of each of those disciplines. Each subject has its own specific vocabulary, and for students to excel in all subjects, they must learn the specific vocabulary of those subject areas *in context*.

Beginning with the middle grades years, students begin to self-select reading materials based on personal interests established through classroom learning. Students become curious about science, mathematics, history, and literature as they form contexts for those subjects related to their personal and classroom experiences. As students explore academic areas through reading, they develop favorite subjects and become confident in their verbal discourse about those subjects.

Reading across curriculum content develops both academic and personal interests in students. As students read, they develop both content and contextual vocabulary. They also build good habits for reading, researching, and learning. The Reading Across the Curriculum standard focuses on the academic and personal skills students acquire as they read in all areas of learning.

MRC. Students will enhance reading in all curriculum areas by:

- a. Reading in all curriculum areas
 - Read a minimum of 25 grade-level appropriate books per year from a variety of subject disciplines and participate in discussions related to curricular learning in all areas
 - Read both informational and fictional texts in a variety of genres and modes of discourse
 - Read technical texts related to various subject areas
- b. Discussing books
 - Discuss messages and themes from books in all subject areas.
 - Respond to a variety of texts in multiple modes of discourse.

- Relate messages and themes from one subject area to messages and themes in another area.
 - Evaluate the merit of texts in every subject discipline.
 - Examine author's purpose in writing.
 - Recognize the features of disciplinary texts.
- c. Building vocabulary knowledge
- Demonstrate an understanding of contextual vocabulary in various subjects.
 - Use content vocabulary in writing and speaking.
 - Explore understanding of new words found in subject area texts.
- d. Establishing context
- Explore life experiences related to subject area content.
 - Discuss in both writing and speaking how certain words are subject area related.
 - Determine strategies for finding content and contextual meaning for unknown words.